Lecture 12

Strong Induction, Tips for Proof Writing

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Remark: Strong induction is more powerful as:

- In "weak" induction to prove P(k+1) we use only P(k).

• In "strong" induction to prove P(k+1) we can use $P(1), P(2), \ldots, P(k-1)$ in addition to P(k).



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If not, then for some $p_i = q_i$, $s/p_i < s$,

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Since $(q_1 - p_1)Q$ is less than s, it should have a unique factorisation.

Say, $s = p_1 \cdot p_2 \cdot \ldots \cdot p_m = q_1 \cdot q_2 \cdot \ldots \cdot q_l$. Notice that every p_i is distinct from every q_i . Assume WLOG $p_1 < q_1$. Let $P = p_2 . p_3 p_m$ and $Q = q_2 . . q_3 q_1$. Consider $s - p_1 Q = p_1 (P - Q) = (q_1 - p_1)Q < s$. Since $(q_1 - p_1)Q$ is less than s, it should have a unique factorisation. Therefore, p_1 should be present in factorisation of Q or $(q_1 - p_1)$.

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AM-GM Inequality
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$$\sqrt[n]{a_1 \cdot a_2 \cdot \ldots \cdot a_n}$$

$$\leq \frac{(a_1 + a_2 + \dots + a_n)}{n}$$

Theorem: Prove that if a_1, a_2, \ldots, a_n are non-negative real numbers, for $n \ge 2$, then

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 $\sqrt{a_1 \cdot a_2} \le \frac{(a_1 + a_2)}{2} \Leftarrow$

$$\Leftrightarrow 2\sqrt{a_1 \cdot a_2} \le (a_1 + a_2)$$
$$\Leftrightarrow 4a_1a_2 \le (a_1 + a_2)^2$$
$$\Leftrightarrow 0 \le (a_1 - a_2)^2$$

Proving P(2):

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$$\sqrt{a_1 \cdot a_2 \cdot \dots \cdot a_k} \le \frac{a_1 + a_2 + \dots + a_k}{k}$$
, for any $a_i \ge k$



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 $\sqrt[2^k]{b_1 \dots b_k \dots b_{k+1} \dots b_{2k}} \leq \left(\frac{b_1 + \dots + b_{2k}}{m} \right)^{2^k}$

$$\overline{a_1 \cdot a_2 \cdot \ldots \cdot a_k} \le \frac{a_1 + a_2 + \ldots + a_k}{k}, \text{ for any } a_i \ge k$$

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 $\sqrt[2k]{b_1 \dots b_k \dots b_{k+1} \dots b_{2k}} \leq \left| \left(\frac{b_1 + \dots b_{2k}}{1 - 1} \right) \right| \leq \frac{b_1 + \dots b_{2k}}{1 - 1} \leq \frac{b_1 + \dots + b_{2k}}{1 - 1} \leq \frac{b_1$

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 $\sqrt[2^k]{b_1 \dots b_k . b_{k+1} \dots b_{2k}} \leq \left(\frac{b_1 + .}{.} \right)^{2^k}$

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$$\frac{\dots + b_k}{k} \left(\frac{b_{k+1} + \dots + b_{2k}}{k} \right) \right]^{1/2}$$
$$+ \frac{b_k}{k} + \left(\frac{b_{k+1} + \dots + b_{2k}}{k} \right)$$



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$$\frac{1}{k} \left(\frac{b_{k+1} + \dots + b_{2k}}{k} \right) \right]^{1/2}$$

$$\leq \frac{\left(\frac{b_1 + \dots + b_k}{k}\right) + \left(\frac{b_{k+1} + \dots + b_{2k}}{k}\right)}{2} \qquad (using 1H)$$

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$$\sqrt[k]{a_1 \cdot a_2 \dots \cdot a_k} \leq \frac{a_1 + a_2 + \dots + a_k}{k}, \text{ for any } a_k = \frac{b_1 + b_2 + \dots + b_{k-1}}{k-1}.$$



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 $\frac{b_1 + b_2 + \dots + b_{k-1}}{k-1} \ge \sqrt[k]{b_1}$

$$. b_2 \dots \left(\frac{b_1 + b_2 + \dots + b_{k-1}}{k-1} \right)$$

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